Preface

Handbook of Cubik Math is a book about problem solving and some of the fundamental techniques used in problem solving throughout mathematics and science. Both the problems and the illustrations of concepts for solving them are drawn from Rubik's Magic Cube.

Ernö Rubik invented the Magic Cube as an aid to developing three-dimensional skills in his students. Little did he realize the impact that this puzzle would come to have. In 1980 alone, approximately five million cubes were sold. Predictions for future years are that sales will continue at more than twice that rate. In almost every neighborhood children – and adults – are playing with the cube.

It certainly enhances three dimensional thinking. However, even greater educational value has been found by mathematicians. For them the cube gives a unique physical embodiment of many abstract concepts which otherwise must be presented with only trivial or theoretical examples. Cube processes are non-commutative – that is, changing the order of movements produces different results. Cube processes generate permutations of the pieces of the cube. Sometimes different processes generate the same permutation so that, by looking at the cube, you cannot tell which process was used. This defines an equivalence between

processes. The concepts of an identity process, inverses, the cyclic order of a process, commutators, and conjugates all playa part in solving problems on the cube. By experimenting on the cube, a student learns about these concepts and their relation to problem defining and problem solving without having to rely solely on his faith in the teacher or the text.

Perhaps surprisingly, one of the most fundamental concepts which cubik math illustrates is the use of symbolic notation. It is extremely rare to find anyone who can master the complexities of the cube without writing down what movements he has made or is planning to make. Without a good symbolic notation this is cumbersome at best. For communicating about the cube with others a common notation is mandatory.

The *Handbook of Cubik Math* in the early technical chapters orients the reader to the basic problem of the cube. It introduces a standard notation – one which is internationally accepted. Then it describes a logical method for restoring any scrambled cube to its pristine state where every face is a solid color. No background of complex or sophisticated mathematical concepts is required in these first three chapters. Many good students in their early teens have mastered these ideas. At the end of Chapter 3, several games are introduced. Playing these will enhance the competitor's understanding of the concepts inherent in controlled modification of the state of the cube.

One might think that after learning how to solve the cube – that is, how to restore it to its monochromatic-sided state – a person would lose interest in the cube. We thought so before we had taught many people how to solve it, only to find that with their increased understanding came increased curiosity. They wanted to understand more about how the cube worked, why processes produced the results they do, and what they could do to enhance their mastery of the cube.

Seldom does one realize at this point that the concepts which appeared so logical for solving the cube problem are, in fact, the concepts of identities, inverses, commutators, and conjugates. Chapter 4 defines these generalized concepts with many examples and exercises from the cube. These principles are applied to derive

new techniques for manipulating the cube. Then in Chapter 5 these improvements are applied to obtain better ways to restore the cube.

It is in Chapter 6 that the mathematical concepts become more sophisticated. It is here that the concept of a group is introduced. The structure and the size of the cube group and its subgroups are explored in Chapters 6 and 7. This leads finally to a discussion of normal subgroups and the isomorphisms of subgroups and factor groups in Chapter 8.

It is expected that some students of the cube will only be ready to absorb material through Chapter 3. Others will be able to work through Chapter 5. The more advanced students will work all the way through to the end. At all stages it is necessary to have easy access to a cube. The cube is the best teacher and experimentation is the best learning technique.